Appendix

Mathematics of Science

Learning Objectives

Upon completion of this chapter, you should be able to:

- Appreciate why mathematics is needed for science
- Be able to perform algebraic operations
- Be able to read a graph
- Know the importance of units and how to convert to SI units

INTRODUCTION

Science is the organization of knowledge into general laws and principles that apply over a broad range of conditions. For example, on the Earth rules may apply, but traveling in space they may be slightly different. This deals with your frame of reference geometrically and in time. Albert Einstein showed time can be distorted, while Isaac Newton treated time as invariant. In other words we might shorten time (and aging) under high-speed interplanetary travel, but on Earth we all age with the same time standard. As we are addressing time mainly on Earth for fire considerations, we do not have to concern ourselves with Einstein's time stretches. However, conditions on the Moon or Mars will subject fire to other gravities, and in orbiting vehicle no gravity is experienced. Scientists are now studying how to expand the material in this book to those non-Earthbound gravitational conditions.

Mass and Energy Concepts

Now let us return to science and only consider the rules that apply on the Earth. First, let us address *mass* and *energy*. While these are common terms, they are not easily defined because they are concepts. Yet or matter contains mass and energy. Suffice it to say that mass pertains to a measure of the quantity of matter. Mass can be considered as that amount of matter expressed as weight; weight being the force necessary to suspend an object of matter in the Earth's gravitational field. Energy might be related to the amount of work it takes to raise an object of matter to a higher elevation on Earth. Work itself is a form of energy defined as the weight of the object times the distance it is raised. Work is a very tangible quantity involving a force (weight) and a distance. Energy in general may not be so tangible. It has many forms, and its identification with work can be simply a way to recognize that it is present in some form. For example, we might have energy due to motion (kinetic energy) or that due the flow of electrons in a wire.

The modern physicist makes no distinction between mass and energy. Indeed, 1 Joule (J) of energy is equivalent to 1.11×10^{-17} kilograms (kg) of mass. This follows according to Einstein who related them through his groundbreaking equation $E = mc^2$. Most have heard of this equation and here we have our first scientific law given as an equation. E stands for energy and m stands for mass- seems logical. c is the speed of light in a vacuum, 299,792,458 m/s (m stands for meter, and s stands for seconds) to be exact. We might also approximate this large number as 300 million m/s. That is the same as 3 with 8 zeroes following it, or 3 times 100,000,000, or 3×10^8. Einstein's equation justifies why 1 Joule (J) of energy is equivalent to 1.11×10^{-17} kilogram (kg) of mass as

$$m = E/c^2 = 1\ \mathrm{J}/c^2 = 1\ \mathrm{J}/(3 \times 10^8\ \mathrm{m/s})^2 = 1/(9 \times 10^{16}) = 1/9 \times 10^{-16} = 1.11 \times 10^{-17}\ \mathrm{kg}.$$

If this is not understandable, hopefully by the time you finish this chapter it will be clearer. First we need an appreciation of mathematics in science, then we need to learn how to manipulate equations into solutions.

In the representation of the speed of light, meters per second were used as its measure. Others might prefer different units as 670,616,629 mph (mph = miles per hour). Ordinary people in the United States would relate to the mph units better, but most scientists use the metric system, or more precisely the Standard International (SI) system of units. Many countries, with the notable exception of the United States, use the SI units in common practice. Now here we see that scientific terms have units, and the units are somewhat arbitrary and vary with tradition and country. But of course, without equivalence between the various units of measure for the same quantity, technological exchange would be impossible. So commerce must rely on precise measurements of mass and energy, and laboratories around the world insure that we have standard scales of reference, and exact conversions between the various different units of measure.

CONSERVATION LAWS

Back to energy and mass. In $E=mc^2$, we see E and m are related through c^2 (c-squared, or c times c). As c is a universal constant (never changing), it means E and m can be exchanged. Of course, they only can significantly exchange under atomic reactions, as in the atomic bomb where the fission of an atom (form of mass) leads to a small loss of mass that produces a big amount of energy. This is seen from the formula, as c is a very large number.

Conservation of Mass

Fire deals with chemical reactions only, so $E=mc^2$ will never practically apply. Chemical reactions re-scramble the atoms of the molecules, and the original mass of the reacting molecules is the same as the molecules formed. The atoms in the molecules at the start and the newly produced molecules at the end remain the same. For example, carbon might react with oxygen and burn to carbon dioxide. If a perfect mixture of carbon and oxygen is introduced (such a mixture is called stoichiometric), none of the carbon and oxygen will remain after the fire. In terms of mass we might consider this as one atom of carbon combining with an oxygen molecule that has two atoms of the element oxygen. The carbon dioxide that forms has exactly these same atoms in its molecule. These original atoms have not been destroyed in the chemical reaction. The matter in terms of the atoms remains the same. As mass is the general measure of the quantity of matter we say: mass has been conserved. This is a general law of nature called the Conservation of Mass. By deduction, as all matter has energy as well as mass, we might conclude that there is also a Conservation of Energy Law.

Conservation of Energy

Energy, however, can take many forms and each has to be recognized and accounted for in applying conservation to energy. For example, a weight that is dropped in a vacuum, offering no resistance, will have all of its *potential energy* converted into *kinetic energy*. If the weight now strikes an insulated immovable plate, the temperature of the weight will rise, as this kinetic energy is now put back into the matter. The temperature is a manifestation of the *internal energy* of the weight. Thus, the internal energy of the weight has increased while its potential energy has dropped an equal amount. In fire, the chemical reaction releases energy in the form of heat. What actually happens in this chemical reaction is internal energy within the original molecules is converted to a new form of energy (heat) as the chemical reaction proceeds. The final internal energy of the molecules that form is less by that amount of heat that was lost. The total energy stays preserved and is redistributed as

Energy of Reactant Molecules = Heat + Energy of Product Molecules.

The difference between the energy of the reactants and products is the energy lost to the surroundings as heat. Thus the sum of energies at the end is the same as the energy at the beginning. This is an example of the Conservation of Energy Law. As no atoms were destroyed or converted into energy in the chemical reaction, the mass is also conserved. Only by re-scrambling the atoms of the reactants into new product molecules has energy been redistributed.

The conservation of energy is commonly called the First Law of Thermodynamics. These Laws for mass and energy cannot be proven; they are based on scientific observations of which no exception has been found. Also "energy" and "mass" are invented terms to express these "concepts" or abstract notions. Thus we need to appreciate that science is the generalization of observations into laws that govern the behavior of the universe. We can relate mass to the quantity of matter (stuff), and all mass has energy that can be recognized in various forms. For example, kinetic energy is defined as mass times one-half of its velocity squared. A weight falling a certain distance is said to do "work" and the rate of doing work is commonly called "power." Water flowing over a dam can be converted from work to kinetic energy of a water wheel that is now spun. But friction robs us of some of this waterpower now generated by the wheel.

By measuring these different forms of energy, the conservation of energy has been validated. In this way, scientists have built up the identification of many forms of energy and have shown that it is conserved when it is transformed to a new form.

Conservation of Momentum

There is one final universal law, and it is known as Newton's Second Law (after Isaac Newton) where

$$F=ma \tag{A-1}$$

Here a is the acceleration of the object mass, m, subjected to a force, F. A mass receiving a fixed force will accelerate according to this formula (law). Equation (A-1) is an expression of the Conservation of Momentum Law as it is called. Momentum is defined as the product of mass and velocity. The equation is given in symbols. These symbols are like acronyms of science. The symbols are used as simplified and shortened nicknames. Often the nicknames have been chosen in an obvious way: F for force, a as acceleration and m for mass. Many scientific texts will use the same symbols for the same quantity. But soon one runs out of the alphabet, as single letters are generally used, and other alphabets, mainly the Greek, are employed. The beginning student in science needs to be aware of such Greek terminology to converse smoothly with equations of science. Table 2-1 lists the upper and lower case letters of the Greek alphabet and their English names. For comfort when dealing with scientific expressions it pays to know some Greek.

Table 2-1. Greek Alphabet Names, uppercase/lowercase

Α/α	Alpha	Ν/ν	Nu
Β/β	Beta	Ξ/ξ	Xi
Γ/γ	Gamma	Ο/ο	Omicron
Δ/δ	Delta	Π/π	Pi
Ε/ε	Epsilon	Ρ/ϱ	Rho
Ζ/ζ	Zeta	Σ/σ	Sigma
Η/η	Eta	Τ/τ	Tau
Θθ	Theta	Υ/υ	Upsilon
Ι/τ	Iota	Φ/φ	Phi
Κ/κ	Kappa	Χ/χ	Chi
Λ/λ	Lambda	Ψ/ψ	Psi
Μ/μ	Mu	Ω/ω	Omega

Let us investigate Newton's Law and consider the meaning of the terms. Acceleration is the rate of change of velocity. For example, if the car speeds up from 0 to 60 mph in 6 seconds, its acceleration is 60 mph/6 seconds or 10 miles per hour per second. It is proper to express this in consistent units, not hours and seconds together. Also if seconds are to be used, generally the foot is a better choice of length over mile. Now we have an exercise in converting from one set of units for length, miles and feet, and one set of units for time, hours and seconds. This can be done in a number of ways. We need to look up how units of one measure are converted to alternative units in that measure. Hours can be changed to seconds (3600 s = 1 hr) and miles can be changed to feet (1 mi = 5280 ft), or we might use 1 ft/s is 0.682 mph.

$$\frac{60 \text{ mph} \times \dfrac{1 \text{ ft/s}}{0.682 \text{ mph}}}{6 \text{ s}} = 14.67 \frac{\text{ft/s}}{\text{s}} \text{ or } \frac{\text{ft}}{\text{s}^2}.$$

Note how (mph) as a unit measure cancels in the numerator and denominator, and we are left with ft/s^2 -- the units of acceleration.

Instead of numbers we could express acceleration in terms of the ratio of velocity to time, or more precisely

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{change of time}}.$$

Symbolically, we could represent a for acceleration, v for velocity, and t for time. Further, We could use the symbol capital delta, Δ, in the Greek alphabet, synonymous with D in the English alphabet, to mean the operation: "change of". Then the previous equation in words can be expressed completely in symbols as

$$a = \frac{\Delta v}{\Delta t}. \qquad \text{(A-2)}$$

More precisely, Newton realized that the interval in time needed in the formula should be very small if he was to compute the precise acceleration of an object as it moves. As a consequence he invented the "calculus" in which Equation (A-2) becomes the "time *derivative* of v" as Δt becomes very small. The acceleration can be expressed as the *instantaneous rate of change* of velocity over time. This book will stop short of the calculus, but one should appreciate that there are higher forms of mathematics that enable more complex and precise computations.

Laws of Nature

In general, the three conservation laws can be expressed in terms of rate equations. These rates are "time derivatives". For example, the rate of energy, $\frac{\Delta E}{\Delta t}$ as Δt gets small is written as $\frac{dE}{dt}$ for its time derivative. The three laws of nature, conservation of mass, energy, and momentum, can be expressed for an isolated system as:

1. $\frac{dm}{dt} = 0$, since the mass does not change over time,
2. $\frac{dE}{dt} = 0$, since energy does not change over time, and
3. $F = m\frac{dv}{dt}$, as now acceleration is expressed as the rate of change of velocity.

These laws, using the calculus, allow scientists to mathematically express the laws of nature. They apply to all processes and must always be satisfied. Consequently if we wish to explain or predict an aspect of any process, we must apply these equations to that process and render a solution. In this way mathematics is the tool of the scientist to describe aspects of nature in terms of concrete numbers, each with appropriate units for a specific quantity. For most applications, this set of three equations cannot be solved exactly to produce a simple formula. Thus, the laws of nature can be expressed in mathematical terms and in conceptual and measurable quantities, but do not easily render specific solutions. Indeed, the process of expressing these laws in complete mathematical terms has evolved over time as more phenomena could be represented.

Scientists began with simple studies to represent a law in mathematical terms. For example, Newton used solid objects (~1700) to first elucidate the conservation of momentum principle. About 60 years later Daniel Bernoulli extended it to water, but he ignored the frictional effects of water flowing through a duct. About another 60 years later Navier and Stokes included this friction. Their description became much more complicated, but the law of nature was the same. The expression of Newton's Law for a flowing fluid, such as water or air, must consider all of the forces present, and must represent the rate of momentum at every point in the flow, and at every time. The Navier-Stokes Equation does this in very complex mathematical terms. In general, the basic three laws of nature that govern all motion, mass, and energy have been expressed in deeper and more complex mathematics over the last 300 years.

Don't get nervous. You will not be asked to solve them. In fact, the biggest computers today can only solve them in very limited ways. We do not yet have the magic universal calculator. Computers solve them approximately. The approximations are not just in the numbers, but also in how to express the unsteady phenomena of fluid turbulence, the rate of the combustion reaction, and the complexities of heat transfer. In addition, the properties of materials are not always precisely known.

Solution by Algebraic Formula

The alternative to precise mathematical solutions is to make observations and measurements, use the laws of nature to the extent possible, and develop formulas for specific, rather than general, problems. These formulas are the results of a solution for limited aspects of a specific process. They have been developed using implications of the conservation laws and careful experimental data so that an algebraic equation gives a result that holds over a wide range of conditions. These are the formulas you will meet in this text.

The algebraic equation will likely have some unusual algebraic operations, at least to the user that has forgotten or never learned algebra. We will review the principles of elementary algebra here.

ALGEBRAIC OPERATIONS AND FORMULAS

The formulas you will encounter in this text will be in terms of elementary algebra. This algebra involves operations with numbers and symbols consisting of normal arithmetic, special functions, powers and roots. We have implicitly been using the algebra to describe the laws of nature so far. But let us backup and become more elementary.

As an example, consider the problem of estimating the time to drive from Washington D.C. to New York City by car averaging 50 mph. The distance is about 230 miles. We can formulate the problem into an equation. Let us choose

v as the velocity of the care

x as the distance

and *t* as the time.

Then,

$$t = \frac{x}{v}$$

or computing

$$t = \frac{230 \text{ miles}}{50 \text{ mi/hr}} = 4.6 \text{ hr}$$

Notice we have the units

$$\frac{\text{miles}}{\left(\frac{\text{miles}}{\text{hour}}\right)}$$

If we multiply this complex fraction in terms of units by $\frac{\text{hour}}{\text{hour}}$ (anything divided by itself is unity or one), we get

$$\frac{\cancel{\text{miles}}}{\frac{\cancel{\text{miles}}}{\cancel{\text{hour}}}} \cdot \frac{\text{hour}}{\cancel{\text{hour}}} = \frac{1}{1} \cdot \text{hour} = \text{hour (hr)}$$

Thus, the answer comes out in the units of hour.

Power Operations

Let us consider another example: What is the area of a track of land 10 meters (m) by 10 m? Let x be the length of one side of the square track, and A stands for area. Our formula is

$$A = x \times x = 100 \text{ square meters.}$$

The symbol $\times$ means "multiply", but here we are multiplying x by itself. Algebra introduces a shorthand for this operation with $x \times x$ as x^2 which means x "square" or to the power 2. Here the power 2, means multiply x by itself.

Let us carry this further. Extending this example to the calculation of volume (V) of a cube have each side of length x

$$V = x^3$$

is the formula. Now we have x^3, or this is called x-cube or x to the power 3, meaning it is multiplied by itself three times. In general, we might write x^n. But what if n is not an integer? That could prove troublesome. Let us investigate this issue.

Roots and Fractional Powers

What if we know that the volume of the cube is 1000 cubic meter (m^3). How can we find the length of a side? We need to find the *cube root* of *V*. This is the inverse of the cube operation or power-3 operation. What number, when multiplied by itself, three times, will give 1000? Of course, we easily see that number as 10, but the cube root of 965 is not something you can do in your head. We could do it by trial and error, and estimate the answer. However, today we are blessed with electronic calculators that can perform all algebraic operations at the stroke of a button. Let us proceed with the language and symbolism of algebra. The operation of taking a cube root is denoted as follows:

$$\sqrt[3]{V} = \sqrt[3]{x^3} = x \,.$$

The cube root has annihilated the cube operation on the *x*, and we return to *x* alone.

Alternative, we can write this root operation as a fractional power

$\sqrt[3]{V} = V^{1/3}$. The 1/3-power is equivalent to the root operation.

Note: $x^3 = x \times x \times x$, so raising it to the 1/3 means undoing the cube operation,

$$\left(x^3\right)^{1/3} = x^1 \text{ or } V^{1/3} = x \,.$$

Thus, we have preformed algebraic manipulation of the formula, by reversing the process of finding *V* from *x*, to finding *x* from *V*.

Scientific Calculators

The operation of having a fractional power implies a root operation. But what if the power is something like, 2.5? Consider 4 to this power. We can dissect its operation as follows:

$$4^{2.5} = 4 \times 4 \times 4^{0.5} = 16 \times 2 = 32.$$

Perhaps that was relatively easy, as we could readily recognize the square root of 4 as 2. If the power was 2.4 instead the answer is lower, and can be found as 27.8576…. Our only recourse is obtaining this number is to use an electronic calculator. These devices can now be purchased at low prices, and can even be downloaded from the web. They are referred to as "scientific calculators". Figure A-1 is an example of such a calculator.

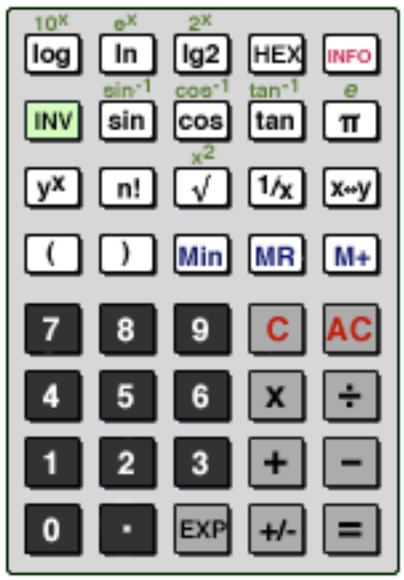

Figure A-1. Typical scientific calculator

The process of computing $4^{2.4}$ is done by using the y^x-key. In this case 4 is typed for y and 2.4 is inputted for x. Pressing the equal button gives the answer. In this class you will need to perform operations like that, but in sequence, such as $4^{2.4}/10.2$. So getting familiar with a calculator is a must. While the precise answer to $4^{2.4}/10.2$ is 2.731…, it could well have been approximated by using $4^{2.5}/10$ giving 32/10 or 3.2. We know it should be lower, and this approximation shows we are on the right track in using the electronic calculator. It is always prudent to do such approximations when performing a series of operations on the calculator.

Negative and Power Zero

Before leaving power operations we should consider what happens if a power is zero or negative. It should be obvious that $x/x = 1$. Writing this as powers, we consider the term in the denominator as a negative power. So the equivalent of $x/x = 1$, can be written as

$$x^1 \times x^{-1} = x^{1-1} = x^0 = 1.$$

Thus we see that the zero power gives 1, and multiplying like terms with powers can be simplified by adding the powers. For example,

$$\frac{x^{3/2}\sqrt{xy}}{y^{2/5}} = \frac{x^{3/2}x^{1/2}y^{1/2}}{y^{2/5}} = x^{3/2+1/2}y^{1/2-2/5} = x^2y^{1/10}.$$

The beginning student of algebra should master these power operations to be better versed at manipulating fire formulas.

Constants in Science

In nature, some "constants" often show up and they are given a special symbol. For example, the area of a circle of radius, r, is

$$A = \pi r^2$$

where π (Pi, the Greek letter) is one such constant. Rounded off to five places,

$$\pi = 3.14159.$$

In Figure A-1 it is seen that the calculator has an individual button for the exact value of π.

Another constant is the quantity called "e" given to five places as

$$e = 2.71828.$$

For the calculator shown in Figure A-1, e shares the same button for π, and can be accessed by pressing a shift key. These are special numbers you are likely to see in the formulas found in fire science.

While π shows up in formulas related to circular effects, it is not obvious why the e shows up. In science, there are many natural processes that, by the conservation laws and the calculus, can be shown to occur with e to some power involving length or time. The expression e^x is called the exponential function of x, meaning the number e is taken to the x-power. This function occurs in a process where a quantity depends on its own rate of change. For example, if the power produced in a fire depends on the rate of change of that power, then with P standing for the firepower, it will behave as

$$P = P_i e^{Ct} \qquad \text{(A-3)}$$

where P_i is the initial value of P, t is time, and C is a constant. If C is negative, the power decreases in time. Of course this happens for fires that die out. On the other hand, if C is positive, the power will continues to increase indefinitely over time. Mathematically this can only reach a very large number at the end of time, called "infinity". Here a mathematical solution gives an answer that is not physically meaningful. With C positive, the fire will grow, but the formula does not take into account the availability of air that will eventually limit this fire growth. We say P grows (or decays) "exponentially" in time.

Logarithms

Consider the exponential decay process. We may wish to compute the time it will take for the power to decay to one-half of its original value. This is commonly called the "half-life". We need an operation that can reverse the power process on the number *e*. This inverse operation is called the "natural logarithm" given by the symbol ln. The process is described as follows:

$$\ln\left[e^{Ct}\right] = Ct. \qquad \text{(A-4)}$$

Logarithms of numbers have been computed and can be found in mathematical tables, but today we can easily access them on the scientific calculator. In Figure A-1 it is seen that there is a button for ln, meaning taking the logarithm of x as $\ln(x)$. Also notice on the same button is the inverse operation of e^x. With this ability we can compute the half-life,

$$\ln(P/P_i) = \ln(1/2) = \ln\left(e^{Ct}\right) = Ct.$$

It can be determined from a table or the calculator that ln(1/2) = - 0.693… roughly. Consequently, the half-life is about - 1.44/C. For this to give a positive number for time, the C must be negative for decay, and must have units of reciprocal time, e.g. 1/seconds or s^{-1}. For example, if $C = -0.01\ s^{-1}$, then the half-life time is 144 s.

This ln implies taking the logarithm to the base *e*. There is another logarithm taken to the base 10 and its operation is denoted by log. Its inverse operation is $\log(10^x) = x$. It too is listed on the calculator.

These special numbers *e* and π, along with the logarithmic operations are likely to show up in scientific formulas.

Graphs

The student rusty in algebra needs to brush up; those that never had it should learn some. But as an alternative to interpreting formulas through algebra, graphs are often employed to show the result. In fact many formulas for fire calculations have resulted from plots of data, and then a formula was developed directly from the graphical fit to these data. So reading a graph is also one way of addressing an equation. For example, a plot of $P = P_i e^{Ct}$ where P_i is taken at 100kW, and C is taken as $0.01 s^{-1}$ is given in Figure A-2.

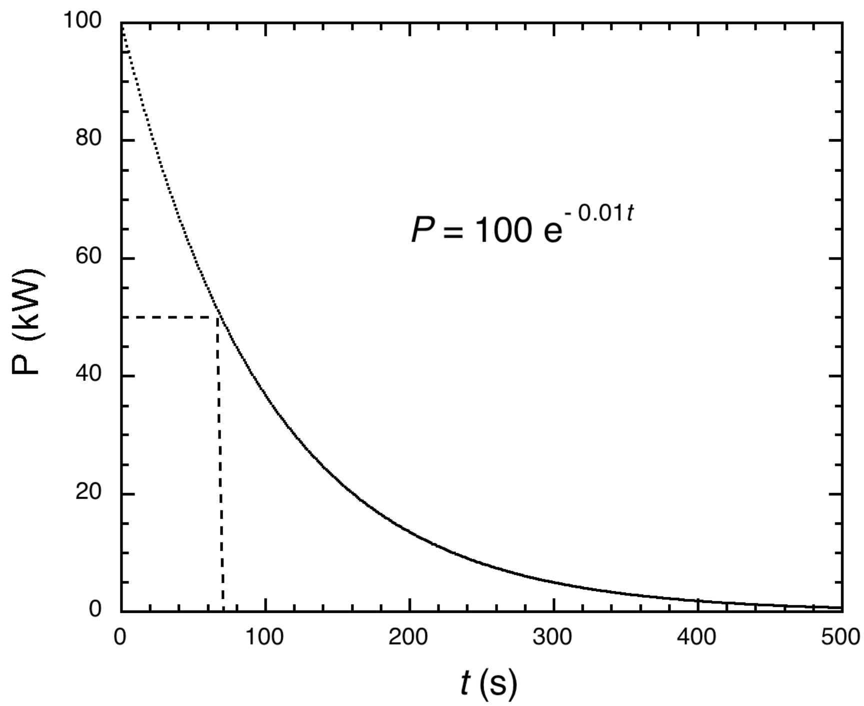

Figure A-2. Linear-scale plot.

In Figure 2-1 a linear scale is used on each of the axes in which the increments on the axes are all equal. On the horizontal axis, the smallest increment is 10 second (s). On the vertical axes, the smallest increment is 5 kW. The graph also shows the half-life solution when the power has decreased to 50 kW. But as the graph approaches zero below a power of about 5 kW, it is difficult to read with any precision. An alternative to reveal more of these low power results is to employ a graph with a logarithmic scale on both axes. This graph is shown in Figure A-3, and it greatly expands the low scale precision. In a logarithmic scale the major increments are broken into decades, and within each decade there are increments inclusive of 1 to 10. These increments within a decade are not equal in length. Therefore, extrapolation of results must be done with care.

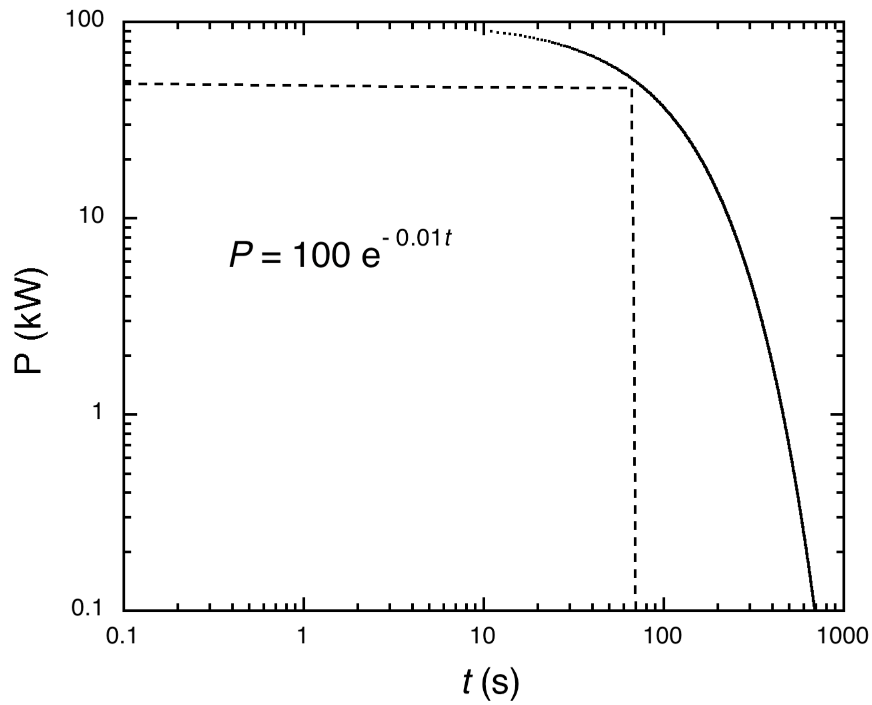

Figure A-3. Logarithmic-scale plot

UNITS AND CONVERSIONS

A formula and a graph will contain various symbols for specific quantities and they will have units. These units must be consistently used or the results will be wrong. If the answer on the left-hand-side is to be in meters, the formula must be algebraically computed from the terms on the right-hand-side to also yield meters. A simple example can illustrate this. Suppose you know the area of a room floor and the length of one of the sides, and you wish to find the length of the other side. You are given that the area is 18.3 m^2 and a side is given as 12.5 ft. These units are inconsistent. Our formula for computing the side, call it s_2, with the given side as s_1, from the area, A is

$$s_2 = \frac{A}{s_1}.$$

Let us maintain meter units, as you will find this more common in science. We need to convert ft to m. Conversion tables are available to making these changes and must be utilized. Table A-2 is a listing of conversion factors that you might find most pertinent to the subject of this course. It emphasizes the conversion of units to the Standard International (SI) system. That system is most often used in science and the formulas of fire are mostly represented relative to SI units. The SI unit for length is m. The other key units are mass in kilograms (kg), and time in seconds (s). All of these units have shorthand symbols for them as indicated. From Table A-2, 1 ft = 0.305 m as equivalent lengths. This equivalence can be used to formulate an algebra for converting, i. e.

$$12.5 \text{ ft} \times \frac{0.305 \text{ m}}{1 \text{ ft}} = 3.81 \text{ m}.$$

In this process, the conversion factor 0.305 was rounded off to three places and the answer has been rounded off to three as well. This too is consistency. Now the problem can be completed in consistent units of m. Substituting into the equation gives

$$s_2 = \frac{A}{s_1} = \frac{18.3 \text{ m}^2}{12.5 \text{ ft} \times \frac{0.305 \text{ m}}{1 \text{ ft}}} = 4.80 \text{ m}.$$

Notice in the algebra of dealing with units that ft cancel, and the m^2 cancels with m to give the length in units of m.

SI Units: Aliases or Derived Units

The conservation laws provide a basis for giving the equivalence of units and minimizing the number of independent units actually needed. However, as in all practices, names spring up to simplify certain collection of units into "aliases". In the SI system we need only deal with kg, m and s as the primary units. Here we have covered mass, length and time. But what units should we give to "force", a distinct quantity? From Newton's Law or the Conservation of Momentum we know force is always equal to mass times acceleration. This means that in terms of the primary SI units we might give force the units of kg times m/s^2. However this name, $kg\text{-}m/s^2$, is cumbersome, so it is given an alias name of Newton (N), as 1 N = 1 $kg\text{-}m/s^2$. This is a so-called derived unit. How about energy? Well energy is measured by work, and work is the product of force times length. Therefore, we can now call energy units in the SI system as N-m. This already has the alias N in it, yet we wish to be simpler, and the alias "joule" (J) is introduced: 1 J = 1 N-m, another derived unit. Finally, power is the rate of energy and is a commodity that is purchased in kilowatts – another alias, the watt (W). 1 W = 1 J/s. In this way many terms in science can be represented in terms of the basic units of mass, length and time: kg, m, and s. These, and other aliases, with their conversions to SI units are listed in Table A-2.

Equations and Dimensions

Let is consider one final example to show how consistency can be maintained. We take an example of a frequently used formula for the ratio of flame length (*L*) to diameter (*D*) for a pool fire given as

$$\frac{L}{D} = 3.87Q^{*2/5} - 1.02 \,. \qquad \text{(A-5)}$$

As *L/D* is the ratio of two length dimensions, flame length to diameter, they must both be in the same units. Also these same units will cancel, leaving no units. The quantity *L/D* has no particular units, and is called as *dimensionless* quantity or variable. Of course the *L* and *D* each have units, and as they must be consistent, they can both be in ft or both in m.

In Equation (A-5), the left-hand-side is dimensionless, and that means the right-hand-side must be the same, dimensionless. The numbers here have no units, and the variable *Q** (Q-star, some have given it the name, the Zukoski number) must also be dimensionless. However, *Q** is a quantity composed of many factors, each having units. Often scientists compose equations that are dimensionless with the intent that they are valid over a wide range of conditions. Also the dimensionless nature of the equation makes it more compact and reduces the number of variables. Here *Q** is composed of the several properties and the variable for the firepower, $\dot{Q}$ in this fashion:

$$Q^* = \frac{\dot{Q}}{\rho c_p T \sqrt{g} D^{5/2}} \qquad \text{(A-6)}$$

where here we have the properties of air given as

Density, $\rho = 1.20$ kg/m^3

Specific heat, $c_p = 1.00$ kJ/kg-K

Temperature, $T = 293$ K.

The SI unit for temperature is in Kelvin (K) and can be considered as an alias for Celsius (ºC). In fact K is a temperature scale that has its zero point associated with no more energy in the material. Celsius is basically a scale of reference for temperature that uses the freezing and boiling points of water as 0 and 100, respectively, instead of 32 and 212 as labeled on the Fahrenheit (ºF) scale. From Table A-2 it can be shown that 293 K is approximately 68 ºF.

One more term is contained in the denominator of Equation (A-6). It is *g*, standing for the gravitational force on a unit mass. For the Earth, this is about 9.81 N/kg. Note as a Newton is an alias, this can be also equivalently written as 9.81 m/s^2. This now has units of acceleration, and *g* is also called the acceleration due to gravity.

Let us compute these terms, as they will be constant for fires on the Earth with the air conditions specified. We note that the units combine as follows when treated in the sense of algebraic factors:

$$1.20\ \frac{\mathrm{kg}}{\mathrm{m}^3}\times 1.00\frac{\mathrm{kJ}}{\mathrm{kg\text{-}K}}\times 293\ \mathrm{K}\times\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)^{1/2}=1101.2\frac{\mathrm{kJ}}{\mathrm{s\text{-}m}^{5/2}}\ \text{or}\ \frac{\mathrm{kW}}{\mathrm{m}^{5/2}}.$$

Here we have recognized the alias: kW for kJ/s. Now if we express the firepower in kW and the diameter in m, *Q** will remain dimensionless and consistent. Often the same equation will be given in specific units. This follows from substituting the air property and gravity term as 1101.2 kW/m$^{5/2}$ in Equation (A-5) to obtain

$$L(\mathrm{m})/D(\mathrm{m})=\left[0.235\mathrm{m/kW}^{2/5}\right]\left[\dot{Q}(\mathrm{kW})/[D(\mathrm{m})]^{5/2}\right]^{2/5}-1.02$$

or

$$L(\mathrm{m})=\left[0.235\mathrm{kW}^{-2/5}\right]\left[\dot{Q}(\mathrm{kW})\right]^{2/5}-1.02D(\mathrm{m}). \qquad \text{(A-7)}$$

Equation (A-7) is the dimensional form of Equation (A-6) in the units of m and kW. Here the units are all indicated, and even the number 0.235 has units, in contrast to the numbers in the dimensionless equation (A-6).

Table A-2. APPROXIMATE CONVERSIONS FROM ENGLISH TO SI UNITS

LENGTH

in	1 inch	25.4 millimeters	mm
ft	feet	0.305 meters	m
yd	yards	0.914 meters	m
mi	miles	1.61 kilometers	km

AREA

in^2	square inches	645.2 square millimeters	mm^2
ft^2	square feet	0.093 square meters	m^2
yd^2	square yard	0.836 square meters	m^2

VOLUME

ft^3	cubic feet	0.028 cubic meters	m^3
yd^3	cubic yards	0.765 cubic meters	m^3

MASS

oz	ounces	28.35 grams	g
lb	pounds	0.454 kilograms	kg

FORCE and PRESSURE

N	newton	$kg\text{-}m/s^2$	alias
Pa	pascal	N/m^2	alias
lbf	pound-force	4.45 newtons	N
lbf/in^2	pound force per sq. inch	6.89 kilopascals	kPa

ENERGY

J	joule	N-m	alias
Btu	British thermal units	1.055 kilojoules	kJ
cal	Calorie	4.187 joules	J

POWER

W	watt	J/s	alias
hp	horsepower	0.746 kilowatt	kW

TEMPERATURE

ºF	Fahrenheit	(ºF - 32)(5/9) Celsius	ºC
K	Kelvin	ºC + 273.15	alias

When an equation is given as a formula, the units are very important. If it is dimensionless to begin with, consistent units must be used for each term. If it is dimensional, specific units must be used for the variables in the equation and nothing other. In Equation (A-7) the units of kW must be used for the firepower, and the units of m for diameter, then the flame length is computed in m also. I was told of the use of Equation (A-7) in a court case where two experts got very different answers. The opposing expert used ft for diameter and got a negative flame length. It may have scored points in that expert's testimony initially, but a lot of credibility was lost on rebuttal. Units are very important. They must be used consistently, or the results of the equation will be wrong. Take care.

Summary

Mathematics is required to solve the laws of science. The key laws of science embrace

1. Conservation of mass,
2. Conservation of energy,
3. Conservation of momentum.

These laws must hold for every natural process, and can be expressed in mathematical terms. Yet even modern computers are not capable for solving all aspects, and can only give approximate answers. Alternatively, science has found specific solutions in terms of single algebraic equations. The solution of these equations into precise numbers requires an understanding of algebra, and the appropriate manipulation of units. SI units are mostly used in science and conversion to these units is essential in many applications. The basic SI units, for purposes of fire science, involve mass in kg, length in m, time in s, and K for temperature. Combinations of these units are sufficient for describing all scientific variables and properties of fire. However, it is practice to give shorthand alias names to the various combinations. This occurs for force (N), energy (J) and power (W). In the computation of any algebraic equation it is essential that the units are consistent so that the units on the right and the left of the equation reduce to the same quantity. A brief review of algebra was presented with emphasis on the use of powers and roots. Exponential and logarithmic functions were discussed. It was also shown how graphical displays of the solution of an equation could be presented in linear and logarithmic scales.

Review Questions

1. What equation shows the equivalence of mass and energy?
2. What equation relates force and mass?
3. How can energy be recognized?
4. What are the basic SI units?
5. Can energy or mass of the universe ever be destroyed?
6. Simplify $\frac{x^{5/2}y^{1/2}e^{2x}}{y^{2.3}\sqrt{x}}$.
7. Compute $\pi e^{0.6}$.
8. Compute the height of a flame in inches from Equation (A-6) for firepower of 18,450 W and a diameter of 60 cm.
9. What force, in N, is required to hold 100 kg in Earth gravitational acceleration of 9.81 m/s^2?
10. Name several derived units in the SI system.

True or False

1. In the chemical reaction of fire, mass is destroyed.
2. Atoms are a form of mass.
3. Heat produced in a fire is due to a rearrangement of the atoms into new molecules.
4. SI units can include feet for length.
5. Newton's law is the conservation of momentum.
6. Units in an algebraic scientific formula must be consistent.
7. Energy is basically just heat alone.

8. The boiling point of water at normal atmospheric pressure is 80 ºC.

9. The units for acceleration are ft/s.

10. Logarithmic scales on graphs are arranged in decades.

Activities

1. Buy or download an inexpensive scientific calculator and learn how to perform simple algebraic operations.

2. Explore tables and conversion techniques found in books or the internet.

3. Explore the conservation laws and their history.

4. Review algebra.

5. Practice manipulating units in scientific equations.